THE EQUIVALENT STATIC ACCELERATIONS
OF SHOCK MOTIONS
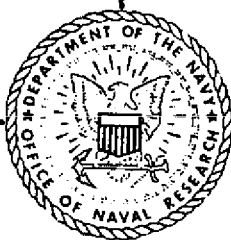
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CONTENTS

ABSTRACT

The use of the reed gage as a method of determining important characteristics of a shock motion is discussed. It is shown that the "shock spectrum," derived from the reed-gage record, provides design conditions for "shockproof" equipment in the form of equivalent static accelerations of the shock motion, and a rational method for comparing the severity of different shock motions.

Shock spectra are presented for certain theoretical motions as well as for the motions of Navy shock-testing machines. Differences between the machines are pointed out and it is noted that the predominant characteristic of the shock-machine motion is a sudden velocity change.

PROBLEM STATUS

This is an interim report on this problem; work is continuing.

AUTHORIZATION

NRL Problem No. F 03-06 R.

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THE EQUIVALENT STATIC ACCELERATIONS OF SHOCK MOTIONS

INTRODUCTION

When contemplating the design of a structure which must be "shock-proof," the designer seeks design conditions which can be used to calculate the sizes of the members of the structure. Because of training and past experience, it is likely that he will attempt to transform the dynamic inertia loads acting on the structure, into equivalent static loads - loads which, if applied statically, will produce similar deflections, stresses, and failures. In the reports of investigations on laboratory shock machines and aboard ship, records of the acceleration imparted to various parts of these structures under shock conditions are found. In general, these records show that the acceleration reached peak values many times the acceleration due to gravity - in some cases thousands of times as much. Knowing that systems can not be designed to carry several thousand times their own weight, the designer chooses some arbitrary number (say 50, 100, or 150) and proceeds on the assumption that the equivalent static load acting on the structure is its static weight multiplied by this arbitrary number.

It is the purpose of this paper to point out: a) that the effect of a given shock motion is not the same for all structures, and b), that the load for which a structure should be designed depends not only upon the shock motion to which it is to be subjected, but also upon its natural frequencies; and to present data useful in the design of structures.

RESPONSE OF SINGLE-DEGREE-OF-FREEDOM SYSTEMS TO SHOCK MOTIONS

The concept of the response of a single-degree-of-freedom system to the shock motion under consideration is of fundamental importance. That this is true will be apparent later. A single-degree-of-freedom-system (Figure 1) is defined as a rigid mass (M) supported by a massless linear spring. The motion or response of the system can be completely described by the displacement of the mass in a direction parallel with the axis of the spring. A
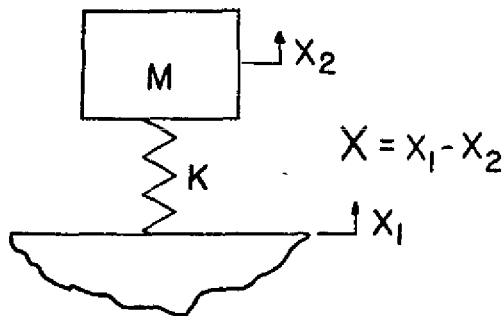
$$X = X_1 - X_2$$

Fig. 1 - A single-degree-of-freedom system

single-degree-of-freedom system has one natural frequency. It is often described as a "simple system" and will be so designated in the remainder of this report. A necessary assumption throughout this report is that the motion (x_1) of the foundation is unaffected by the motion of the structure which it supports.

There are several methods of finding the response of a simple system (Figure 1) to shock excitations. If an analytic expression for the shock excitation is known, the response of the simple system can be obtained by Duhamel's integral.[1] For each natural frequency assigned to the simple system, a response to the excitation can be computed.

When the excitation is given only in graphical form (an acceleration time curve taken aboard ship for example) there are two convenient methods of determining the response of simple systems to such excitations. The first is a graphical method of evaluating Duhamel's integral, described in detail in reference[2]. By this method, the maximum value of the acceleration of the mass (M) can be found for any natural frequency of the system when x_1, the acceleration of the base of the simple system, is given as a graphical record or an analytical expression which can be plotted.

The second method is by use of a torsion pendulum. Biot[3] has shown that if the support of a torsion pendulum (Figure 2) is given an angular displacement θ proportional to $\ddot{x}_1$ (the acceleration of the base of the simple system (Figure 1) then ϕ (the absolute angular displacement of the bob of the torsion pendulum) is proportional to the relative displacement (X) in the simple system. In the discussion of the design of earthquake-proof buildings, Biot found the maximum value of the acceleration of the mass of a simple system $(\ddot{x}_{2\ max})$, subjected to a given earthquake acceleration-time record. This maximum acceleration was plotted against the natural frequency of the simple system, and this plot was called an "earthquake spectrum."
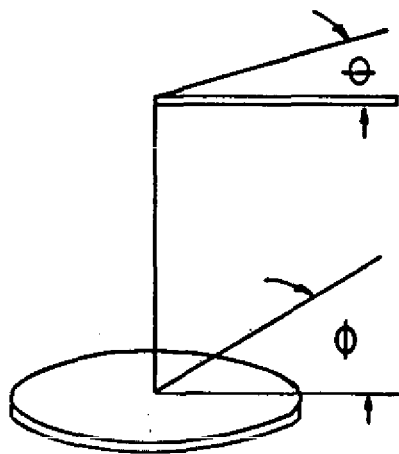

Fig. 2 - Diagram of torsion pendulum

If a series of simple systems could be constructed and subjected to an actual shock motion, and if the relative motions (X) were recorded, a "shock spectrum" could be plotted similar to the earthquake spectrum. The reed gage (Figure 3), designed at the David Taylor Model Basin approximates such an instrument. The reed gage, as its name implies, is composed of a series of cantilevers carrying masses at their free ends. Each cantilever has a different natural frequency in the first mode. The maximum value of the displacement of the free end, relative to the fixed end, is recorded by scribing on waxed paper.

THE DERIVATION OF SHOCK SPECTRA FROM REED-GAGE DATA

In order to obtain the shock spectrum of a given shock by use of the reed-gage, the excitation is applied to the

[1] T. Von Karman and M. A. Biot, Mathematical methods in engineering, (McGraw Hill, 1940), p. 403.

[2] G. E. Hudson, A method of estimating equivalent static loads in simple elastic structures, David W. Taylor Model Basin Report No. 507, June 1943.

[3] M. A. Biot, Bulletin of the seismological society of America, 31: 151-171, (1941)

base of the gage, and the maximum deflections of the reeds relative to the base are recorded.

If the reeds are considered to be simple systems, for each reed there is a static deflection δ_s (due to the action of gravity alone) which is related to the natural frequency f_n of the reed by the familiar equation

$$f_n = 3.13 \sqrt{\frac{1}{\delta_s}} \text{ cps} \tag{1}$$

Let X be the deflection of a reed tip relative to the base of the instrument under shock.

$$\text{Then} \qquad \frac{X_{max}}{\delta_s} = N \tag{2}$$

Where N is the "equivalent static acceleration." The equivalent static acceleration is the gradually applied acceleration, expressed as a multiple of the acceleration due to gravity, to which the reed must be subjected in order to produce the same deflection of the tip as was produced under shock.

This value of N is plotted against the primary natural frequency of the reed. This plot of N against f_n is called the "shock spectrum" of the shock to which the reed gage was subjected.

EXAMPLES OF SHOCK SPECTRA

Figures 4 and 5 show the shock spectra for some simple motions which can be expressed and analyzed mathematically. These and many other similar curves have been published by several writers.

Fig. 3 - Reed gage, (cover removed), showing reeds and records scribed on wax paper
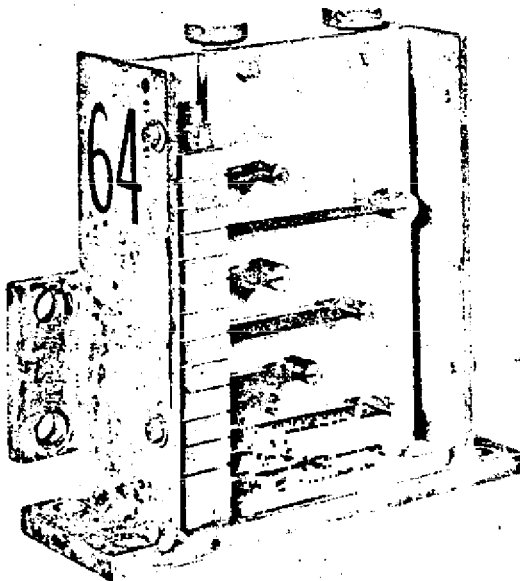
Curve 1, Figure 4, is the shock spectrum for a half-sine pulse of acceleration defined by equations (3).

$$\ddot{x}_1 = a \sin\frac{2\pi t}{T} \quad \text{for} \quad 0 \leq t \leq \frac{T}{2} \quad \text{and}$$

$$\ddot{x}_1 = 0 \qquad \text{for} \quad t < 0 \text{ and } t > \frac{T}{2} \tag{3}$$

N/a is the ratio of the peak acceleration of the mass of a simple system to peak acceleration, a, of the applied shock motion and f_n is the natural frequency of the simple system. Assigning values to a and T reduces the coordinates to the conventional ones (i.e., N and f_n).

Curve II shows the effect on the shock spectrum of having the sine pulse of curve I
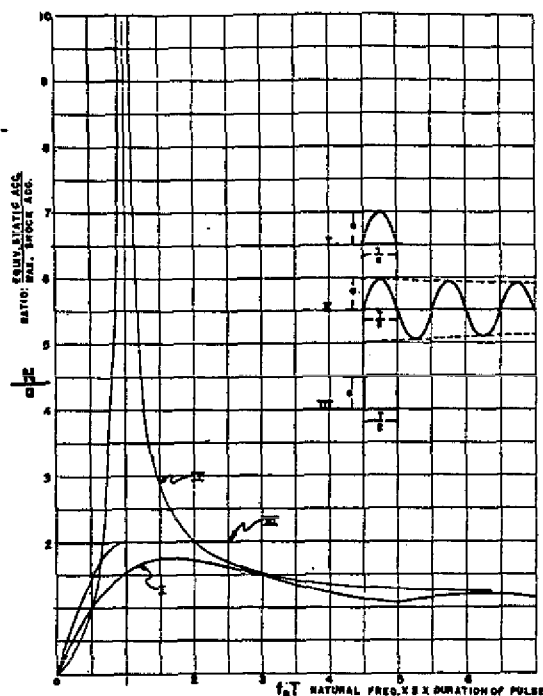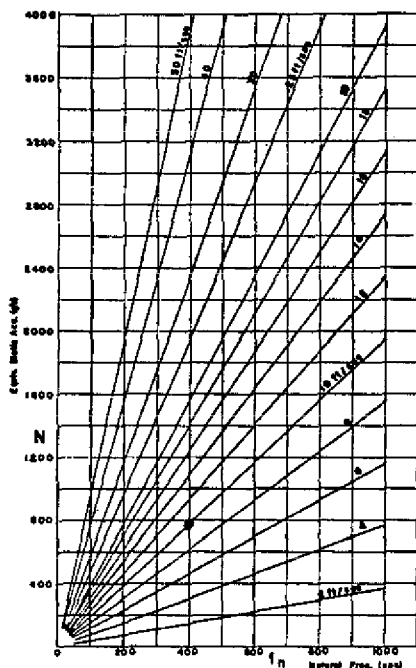
Fig. 4 – Shock spectra for
three simple shocks

Fig. 5 – Shock spectra for
velocity change

continue on as a damped transient vibration. This motion is defined by

$$\ddot{x}_1' = 0 \qquad \text{for } t < 0$$

$$\text{and } \ddot{x}_1 = e^{-bt} \, a \, \sin\left(\frac{2\pi t}{T} + \gamma\right) \text{for } t \geqq 0 \quad (4)$$

In the curve shown, b is appropriate for 0.5 percent of critical damping and the phase angle γ is practically zero. This curve is very similar to a steady-state vibration transmissibility curve.

Curve III is the shock spectrum for a "square" pulse of acceleration defined by

$$\ddot{x}_1 = 0 \text{ for } t < 0, \ t > \frac{T}{2} \text{ and}$$

$$\ddot{x}_1 = a \text{ for } 0 \leqq t \leqq \frac{T}{2} \qquad (5)$$

Figure 5 is the shock spectrum for a step velocity change defined by

$$\dot{x}_1 = 0 \text{ for } t < 0 \text{ and}$$

$$\dot{x}_1 = V \text{ for } t > 0 \qquad (6)$$

Various values of the velocity change (V) have been used to obtain the family of curves shown. The equation of this family of curves is given by

$$N = \frac{2\pi f_n V}{g} \qquad (7)$$

SIGNIFICANCE OF SHOCK SPECTRA

Shock spectra are important because they can be used to calculate whether or not a given shock will damage a specific structure. In order to use the shock spectra in designing a structure, it is necessary to assume that the structure is elastic and linear up to the point of failure, and is subject to the same shock motion as the reed gage.

The kinds of damage to a structure which are predictable from the use of shock spectra are: the yielding of ductile materials, the fracture of brittle materials, and the collision of adjacent parts through elastic distortion of their supports. Types of failure not predictable from shock spectra include: breaking of a friction bond, as in the loosening of screws or opening of knife switches; spilling of liquids; fracture of ductile materials; and collision of unsecured parts.

When a structure can be assumed to be a simple system (i. e., a single degree of freedom system) and

the shock spectrum is available, it is an easy matter to calculate its natural frequency and to determine the stress and deflection which will be caused by the shock. This calculation is simple because the shock spectrum gives the equivalent static acceleration from which the equivalent static load may be computed. Since static stress is proportional to static load, the shock spectrum enables one to predict stress, and hence damage.

Similar calculations of stress and deflection are possible for a multi-degree-of-freedom structure if the flexible members of the structure are linear and elastic. The method of calculation is given in reference (4) and will be discussed in detail in a future report. The important parts of the method are the following: First, it is necessary to find the shapes of the modes of vibration of the structure, and the natural frequencies associated with them. The amplitude of each mode excited by a shock motion is proportional to the number of g in the shock spectrum corresponding to the natural frequency of the mode. Since each mode is analogous to a simple system, the total stress and deflection of the structure can be found by superposing the response of the various modes.

The shock spectrum is an excellent index of the damaging capacity of a shock, for if the vulnerability of a structure to two different shock motions is under consideration, the spectrum with the greater value at the natural frequency of the structure is the one which will produce the greater stress or likelihood of damage. The shock spectrum is also an excellent index of the relative protective abilities of two or more shock mounts. One shock mount is considered the best of a lot in the performance of its protective function, if the motion of a mass mounted on it has, during shock, a lower shock spectrum than that of any of the others.

The shock spectrum presents a shock motion in a graphical form which can be easily understood. On the other hand, by looking at the velocity-meter records, Figures 11 and 12, one can see that the motion at the center of the 4A plate is different from that at the edge; but it is not apparent what the effect of this difference will be upon structures to be tested. The shock spectra Figures 6 through 10 show the significance of this difference, which will be discussed in the following section.

NAVY SHOCK-MACHINE SPECTRA

Reed-gage records were obtained on the Navy Light-Weight Shock Machine, prior to its modification in 1947, by mounting the gage on the 4A plate and subjecting the anvil plate to 1 ft, 2 ft, 3 ft, 4 ft, and 5 ft back blows. The shock spectra obtained of the motion of the center of the 4A plate are shown as dotted lines in Figures 6 through 10. The reed gage was also placed at the edge of the 4A plate directly over the channel support of the 4A plate. The shock spectra of the motion at this location are shown in Figures 6 through 10 as solid lines. These spectra were obtained by connecting the observed points with straight lines. The motion at the two locations on the 4A plate as recorded by a British type velocity meter are shown in Figures 11 and 12. Peaks occurring in the shock spectra are due to near-resonance of particular reeds with transient vibrations of the anvil and 4A plate. The width and height of these peaks are not well defined because of the small number of reeds used. Since the frequency of the transient vibration probably did not coincide with any reed frequency, the true peaks are probably higher than shown and resemble, in shape, the peak of Figure 4, curve II.

Reed-gage records obtained on the Navy Medium-Weight Shock Machine are shown in Figures 13 through 15 The important features of the motion of the anvil plate of this machine

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[4] M. A. Biot, Natl. Acad. Sci. (U.S.) 19: 262-68 (1933)

Figures 6 through 10 indicate shock spectra for the Navy Light-Weight Shock Machine. Dotted line-center of 4A plate. Solid line edge of 4A plate over channel.
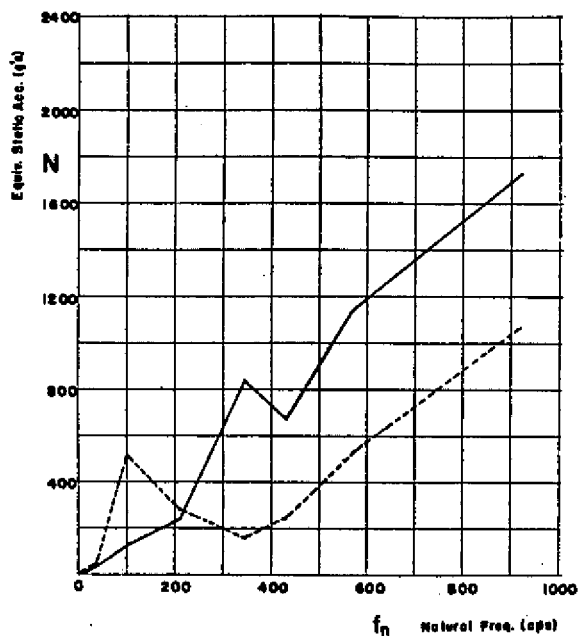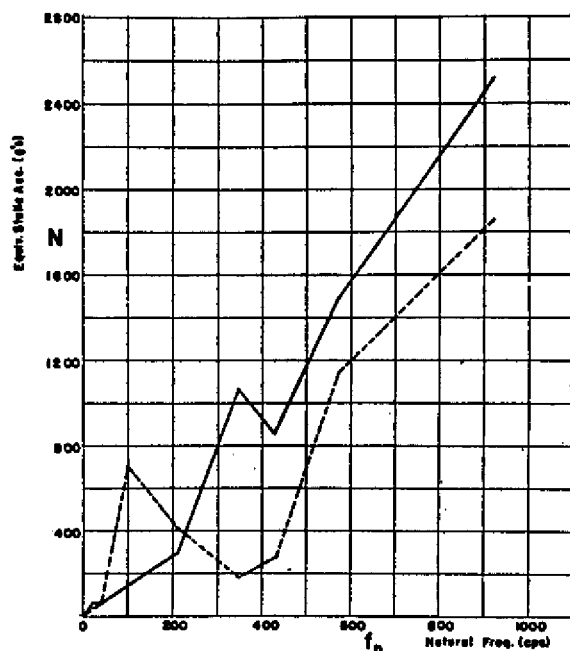

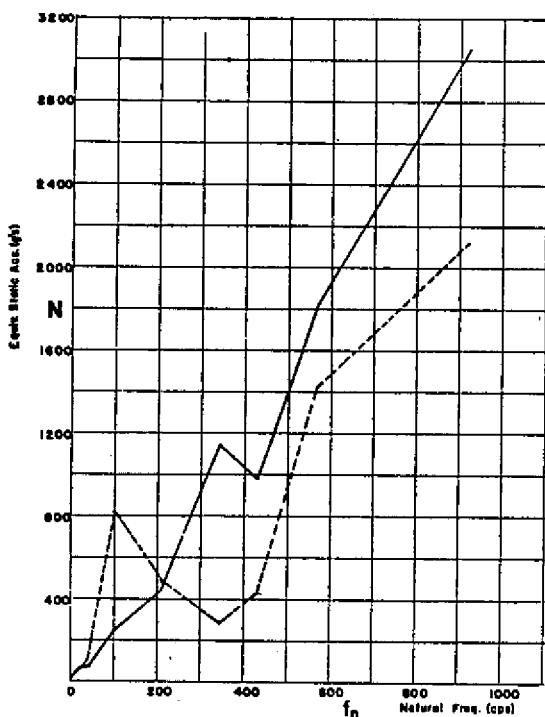Fig. 6 - 1-foot back blow

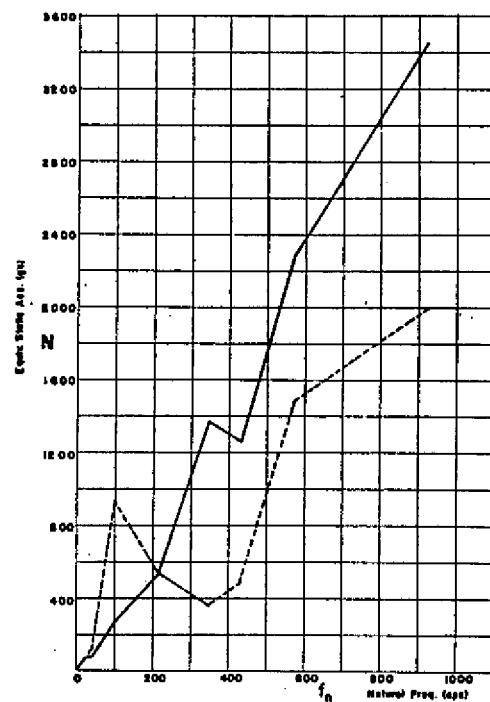Fig. 7 - 2-foot back blow

Fig. 8 - 3-foot back blow
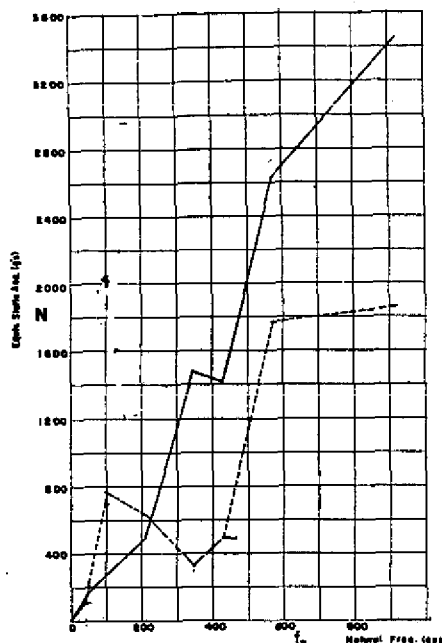

Fig. 9 - 4-foot back blow

Fig. 10 - 5-foot back blow

are two successive velocity changes - one due to the impact of the hammer, the second to the impact of the anvil plate against rigid stops. The effect of the second impact (velocity change) upon the reeds (already vibrating from the first impact) depends upon the phase of the reeds at the moment of the second impact. The amplitudes of some reeds are increased by the second velocity change while those of other reeds may be unaffected or decreased; hence the record shows some deviations from the straight-line characteristic of an instantaneous velocity change.

A comparison of Figure 5 with the shock spectra of the shock machines shows that the predominate characteristic of the shock-machine motion is an instantaneous velocity change. These records demonstrate that designing for some fixed value of g without regard for the natural frequency of the structure being designed is not correct and cannot be justified. When it is desired to reduce the description of a shock machine motion to a simple equivalent, this can be done with less error by finding the velocity change the spectrum of which lies just over the shock spectrum.

The spectra of the motion of the center of the 4A plate on the Navy Light-Weight Shock Machine shows clearly an important characteristic of the motion at this point—the relatively high "equivalent-static-accelerations" to which the 100 cps systems are subjected. This high acceleration is a result of the transient
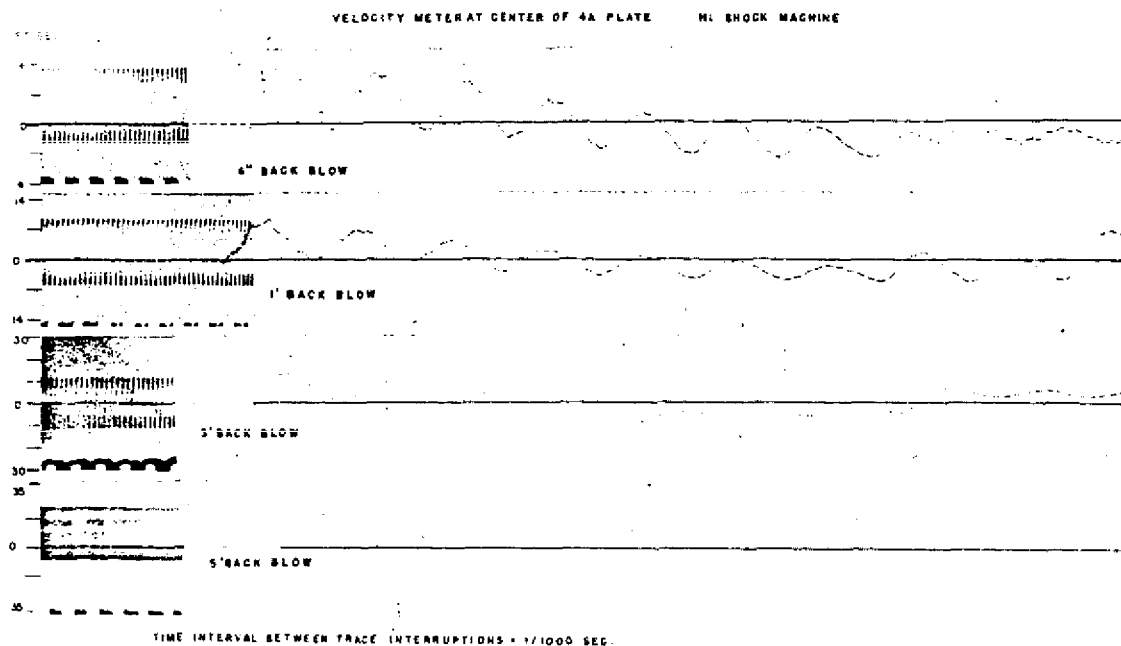

Fig. 11 - Velocity - time record of motion of center of
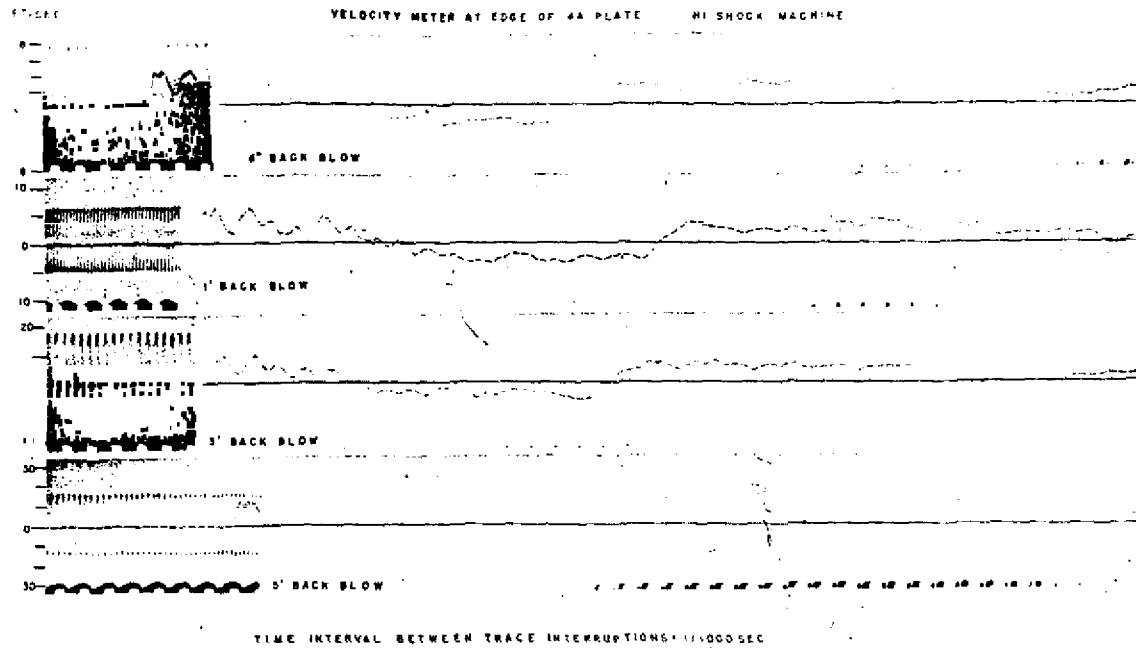4A plate Navy Light-Weight Shock Machine

Fig. 12 - Velocity - time record of motion of 4A plate over
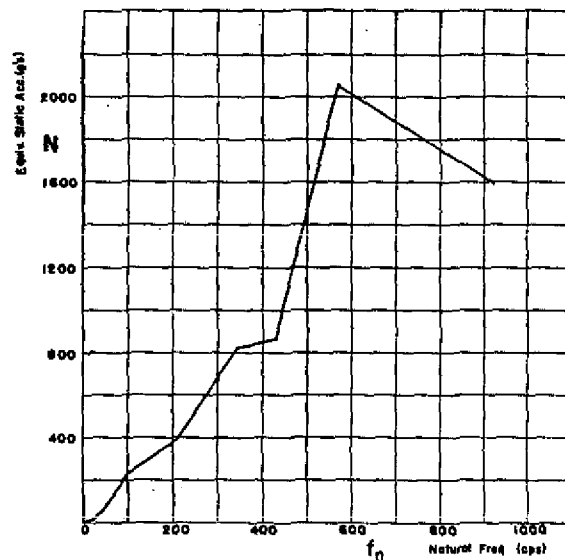supporting channel Navy Light-Weight Shock Machine

Fig. 13 - Shock spectra of the motion of the
Medium-Weight Shock Machine, 3-inch table
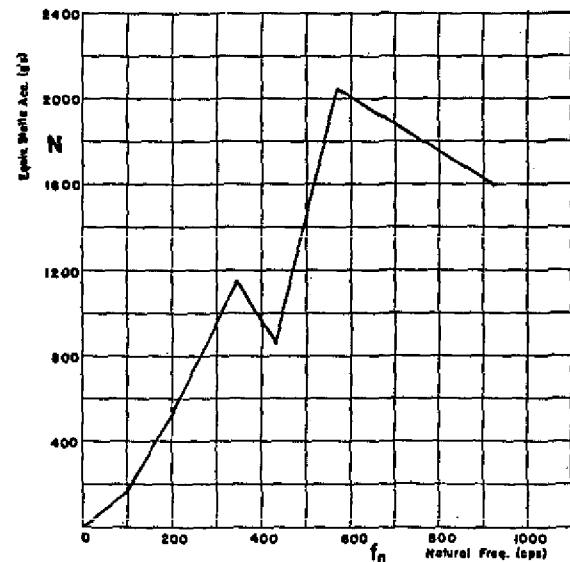travel, 1-foot hammer drop, 500-pound load
on table

Fig. 14 - Shock spectra of the motion of the
Medium-Weight Shock Machine, 3-inch table
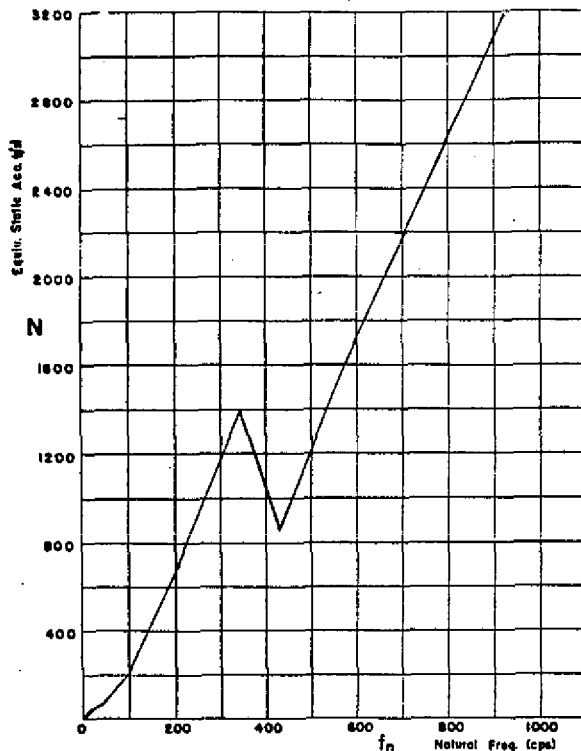travel, 2-foot hammer drop, 500-pound load
on table

Fig. 15 - Shock spectra of the motion of the Medium-Weight Shock Machine, L5-inch table travel, 1-foot hammer drop, 500-pound load on table

vibration of the 4A plate, which is shown on the velocity-time curve of this motion, (Figure 12). It has been pointed out earlier that the maximum height of the peak in the vicinity of 100 cps is not known exactly. However, this peak, if found exactly, would have a shape similar to curve II of Figure 4. It has been shown, then, that the Light-Weight Shock Machine, because of the vibration of the 4A plate, can cause systems having a natural frequency in the vicinity of 100 cps to be subjected to at least five times the acceleration of systems having a natural frequency of 50 cps. The shock spectra for locations at the edges of the 4 A plate over the channels do not exhibit this peak at 100 cps; neither do the shock spectra for the Medium-Weight Shock Machine. When the response of systems having approximately 350-cps natural frequencies are examined, other marked differences are seen. At the center of the 4A plate on the Light-Weight Shock Machine, the equivalent static acceleration of the 350-cps system has about the same value as the 50-cps system, several times less than that of the 100-cps system. However, the equivalent-static-acceleration for the 350-cps system on the Medium-Weight Shock Machine or over the channels on the Light-Weight Shock Machine, has continued to rise from the value for the 100-cps system with the general characteristic of an instantaneous velocity change.

Now since the spectra are measures of the damaging effect of a shock motion, they demonstrate that different results from the shock test machines can be obtained, depending not only on which type machine is used but, also, in the case of the Light-Weight Shock Machine, where the component is mounted.

SUMMARY

The importance of the "shock spectrum," which can be derived from a reed gage record of a shock motion, has been discussed. It has been pointed out that the shock spectrum of a shock motion gives the gradually applied accelerations which will produce deflections and stresses in a simple system equal to those produced by the shock. These equivalent static accelerations are a function of the natural frequency of the simple system and are the proper basis for design of structures to withstand shock. The practice of designing structures for a single, arbitrarily chosen, equivalent static acceleration, without regard to natural frequency, is shown to be incorrect by presenting shock spectra obtained on Navy High-Impact Shock Machines. These spectra show that the best approximation of the shock machine motion is a sudden velocity change.

Shock spectra give a unique basis for comparing the relative damge capacity of different shock motions. Shock spectra are being used to study the performance of shock mounts.

Comparison of the shock-machine spectra shows that the damage capacity of the two types of Navy Shock Machines differ and that in the case of the Light-Weight Shock Machine significantly different spectra are obtained for different locations on the test-specimen mounting fixture.

APPENDIX I

NOTES ON THE REED GAGE

The present reed gage (Figure 3) diverges from the theoretical ideal in several respects. Corrections can be made for some deviations on a theoretical basis, but other errors are inherent in the instrument. The following observations are the result of experience in using and applying the reed gage.

1. The accuracy with which a deflection can be measured is not satisfactory for the high frequency reeds. It is believed from experience, that reed deflections cannot be measured closer than 0.01 inches. This can amount to an error of the following number of "g" depending upon the frequency.

Reed Natural Frequency	Error (in g's)
20	.40
40	1.39
100	8.82
210	37.70
345	120
430	185
570	316

These errors are due largely to the use of rounded point styli on waxed paper to record deflections. The line scribed is broad and poorly defined. Suggested improvements have not yet obtained the desired simplicity, freedom from friction drag, and ability to operate during shock. There is also an error because the scribed deflection mark is an arc rather than a straight line. This is not important, since it predominates in the low frequency reeds and is a small error in terms of equivalent static acceleration. Deflection is customarily measured as the perpendicular distance from the end of the arc to the zero-deflection line. Occasionally, shifting of the recording paper, when it is removed from under the reeds after a shock, makes the zero position of the reeds difficult to locate.

2. The mass of an actual reed is distributed rather than concentrated at the tip. The proper point at which to measure the deflection is not the center of the weight at the tip of the reed, but some effective center of mass. It is possible to find the ratio of the deflection of a reed at the point measured, to the deflection which a simple system of the same frequency would undergo.[5] The following table gives numbers, corresponding to standard reeds of the gage, which should be multiplied by the measured reed deflections to find the deflections which a simple system would have recorded.

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[5] E. Z. Stowell, et al. Bending and shear stresses developed by the instantaneous arrest of the root of a cantilever beam with a mass at its tip. NACA MR No. 14K30

Reed Frequency	Correction Factor
20	.97
40	.845
100	.860
210	.93
345	.98
430	.97
570	.945

The above effect of distributed mass is quite different from the effect of higher modes of vibration of the reeds. The deflection of the reed tip due to the second and higher modes is, except in rare cases of resonance, a negligibly small fraction of the tip deflection due to the first mode.

3. Friction of the scribers on the waxed paper causes damping of the reeds. This is certainly not the same amount and type of damping to be found in the practical structures which the reed gage is supposed to represent, though it would be difficult to specify an ideal damping characteristic. Examination of the transient, traced upon a special moving paper, as a result of a velocity-change shock shows the damping to be coulomb (rubbing) friction. The decrement of amplitude per cycle was uniform regardless of amplitude and is tabulated in the following table.

Reed Frequency (cps)	Decrement Per Cycle (inches)	Decrement Per Cycle (g)
20	.132	5.28
40	.115	16.0
100	.014	12.4
210	.004	15.1

For shocks which will build up the simple system amplitude by resonance, damping will cause a large error. Except in such cases, the error is negligible. In the case of resonant build-up of vibration, it is not yet clear whether or not a simple undamped system represents an actual structure any better than the friction-damped reed. If a practical structure is to fail by successive increases in its amplitude of motion because of resonance with a transient vibration in the shock motion, it must, on some cycle, pass from wholly elastic deformation into a slight plastic deformation. This orovides a considerable increase in the damping and may thus prevent the deformation from exceeding an acceptable value. Further investigation of this important question is required.

4. The number of reeds which can be accommodated in the reed gage frame is too small to define satisfactorily a shock spectrum. The present gage is already too large for some applications. The difficulty is largely inherent in the nature of the gage since a large space is required for motions of the low- frequency reeds. Shorter low-frequency reeds would be more non-linear than the present reeds since their deflection would be a larger fraction of their length, and since lighter reeds are affected more by scriber friction. Perhaps the solution lies in having several reed gage designs - each for a special type of application.

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